KINEMATICS FUNDAMENTALS

Introduction

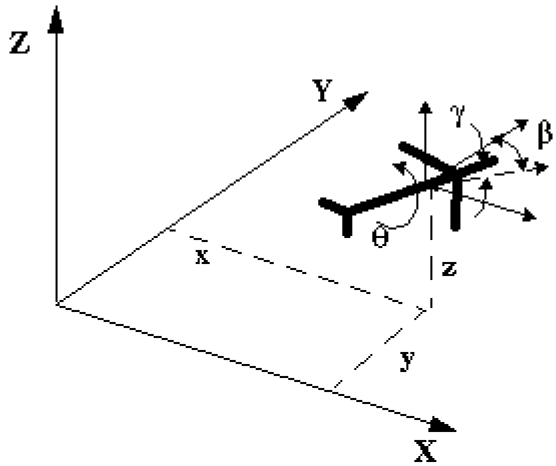
One of the topics that we will be studying in EM477, Computer-Aided Design is mechanism design. The objective of this section is to lay the foundation for our study of this subject by providing you with some definitions and concepts that will be used in our later discussions. We will be investigating both the *kinematics* and *kinetics* of mechanisms so let us start by defining these two terms.

Kinematics - the study of the motion of assumed rigid bodies, without regard for the resulting forces. A complete kinematic analysis will enable us to know the position, velocity and acceleration of any point on a mechanism as a function of time.

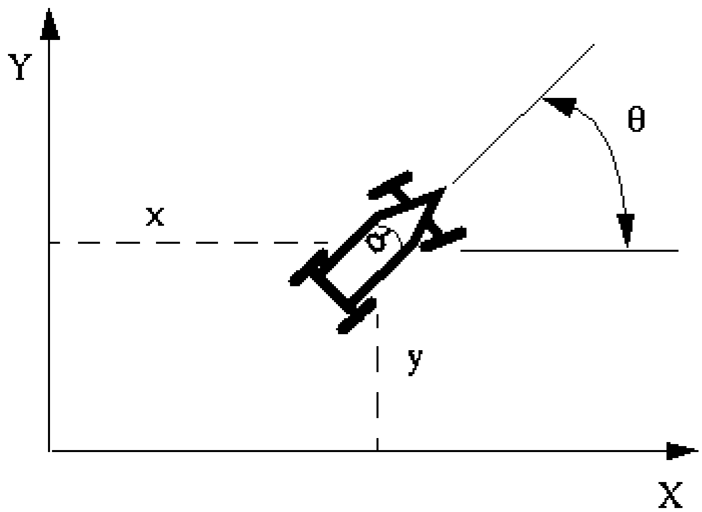
Kinetics - the study of the forces that result from, or lead to, the motion of rigid bodies. A kinetic analysis will let us know the internal and external forces that are developed in a mechanism as well as tell us how much power we must provide to operate a mechanism to perform a particular function. This information is essential in order to conduct the detail design of the individual components of the mechanism.

We will spend the first few weeks studying the kinematics of mechanisms and then take a look at the kinetics.

In order to define the exact position of an object in space at some given instant of time, we must specify a sufficient number of parameters that will uniquely and unambiguously locate the object with respect to some reference frame. Consider an airplane flying through the air. If we want to specify the exact position of the airplane with respect to a reference frame located at the airport, how many parameters would we have to specify? Three? Five? Six? We would need to specify a minimum of six components to unambiguously define the position of the airplane. Three of these components could be the x, y and z coordinates of the center of gravity (CG) of the airplane with respect to the reference frame. The other three components could be three angles which describe the attitude of the airplane with respect to a local coordinate system located at its CG such as the pitch, yaw and roll angles. This set of six parameters is not the only set that would suffice to locate the airplane, there are an infinite number of combinations of parameters that would do the job. The important point is that we need a *minimum* of six parameters. We will define the *degrees of freedom* of an object as the number of independent parameters needed to define the position of the object in space.



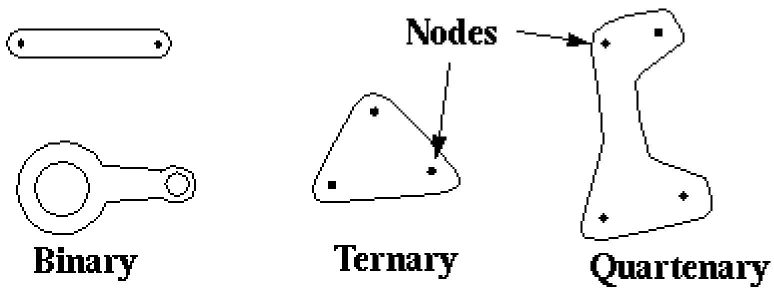
What if we restrict an object to lie in a specific plane? Think about a car driving on a parking lot. How many parameters are needed to specify the position of the car with respect to a corner of the parking lot? The correct answer is three. We could specify the x and y coordinates of the left front tire and the angle of the centerline of the car with respect to one of our coordinate axes.



In general, an object permitted to move in 3-D space has six degrees of freedom (DOF) and an object restricted to 2-D planar motion has three DOF. In order to analyze a mechanism, one of the first steps is to determine the DOF of the mechanism.

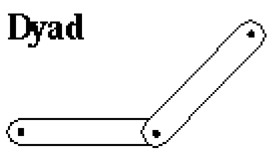
If we are going to study mechanisms, it would help to define exactly what we mean by a mechanism. Before we do that however, we need to cover a few more terms.

link - assumed rigid body with two or more nodes (attachment points). A link can be classified by the number of nodes it contains. A binary link has two nodes, ternary - three nodes, quartenary - four nodes, and so on.

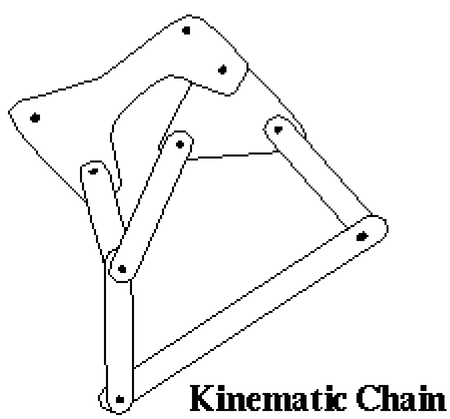


joint - a connection between two links which allows some relative motion. A joint can be classified by the number of DOF it permits. A pin joint between two links has one DOF (rotation). Another single DOF joint would be a square pin in a slot.. The DOF would be the position of the pin within the slot with respect to one end of the slot. An example of a joint with two DOF would be a round pin in a slot. The pin can translate along the slot as well as rotate about its own axis.

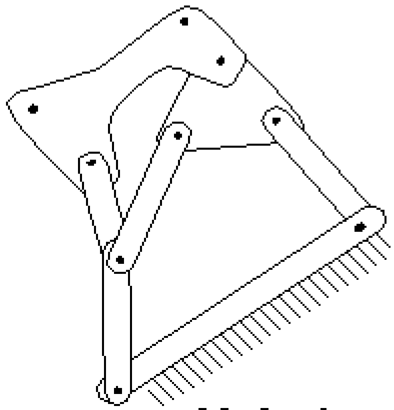
Dyad - two links connected by a joint



kinematic chain - a collection of links and joints connected in a manner to provide a controlled output in response to a supplied input.



Mechanism - a kinematic chain that has had one line rigidly attached to the reference frame (ground).



Mechanism

Machine - a collection of mechanisms arranged to transmit forces and do work.

Now that we understand what a mechanism is, we can define the degrees of freedom of a mechanism as the number of inputs needed to define the output state. In the designing a mechanism, you have some idea of your desired output. You want to achieve the output with a minimum number of inputs. Each input (i.e., a motor or actuator) requires a controller, adds weight and cost to the system, requires maintenance and is a potential source of breakdown. Therefore, it is desirable to design mechanisms that require as few inputs as possible, ideally a mechanism has just one DOF to achieve the desired output. Of course this is not always possible, particularly for mechanisms that must perform highly complex functions, but many useful mechanisms have a single DOF.

Greubler's Equation

So, how does one go about determining the DOF of a mechanism? For this discussion and most of this course, we will be restricting our study to 2-D mechanisms. Therefore, if we consider a link lying in a plane, it has three DOF. If we have **L** links, then the collection of links has 3×**L** DOF. If we have two links (**L**=2) in a plane, then we have 3×2=6 DOF.

If we use a full joint, such as a pin joint, to connect the two links together, then we remove 2 DOF and the system only has 4 DOF. Every full joint added to the system removes 2 DOF. A half joint such as a roll-slide joint removes one DOF.

If we ground a link by attaching it to the reference frame, then we remove all of that links DOF, so for each grounded link in the mechanism, we remove 3 DOF. So **G** grounded links remove 3×**G** DOF from the system. Now grounding more than one link is the same as combining the links into a single, large link and grounding that, so in general, there is only one ground link **G=**1.

We can put these ideas together and use them to formulate a general expression for determining the number of DOF of a mechanism.

DOF = 3**L** - 2**J** - 3**G**

where **L** is the number of links, **J** is the number of joints (full joints count as one, half joints count as one-half) and **G** is the number of ground links, but since all ground links can be combined and counted as one, **G**=1 resulting in:

DOF= 3(**L**-1) - 2**J**

which is known as Greubler's equation. If **DOF** = 0 then you do not have a mechanism, but a structure. If **DOF**=-1, then you have a pre-loaded structure (i.e. it can't be assembled without stressing at least one of the links).

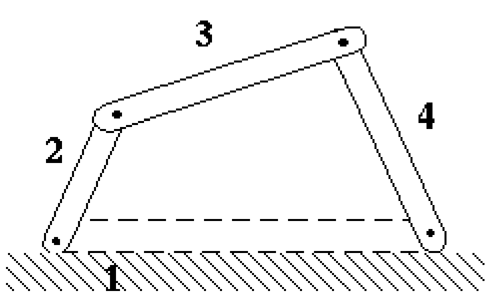
Example - Consider the mechanism from the previous section. How many DOF does this mechanism have? First, count the number of links - there are seven, so **L**=7 (don't forget the ground link). This mechanism has eight joints, so **J**=8 (if more than two links connect at a node, then the number of joints increases by one for each additional link at the node beyond two). Applying Greubler's Equation with **L**=7 and **J**=8,

DOF=3(7-1) - 2(8)
DOF=2

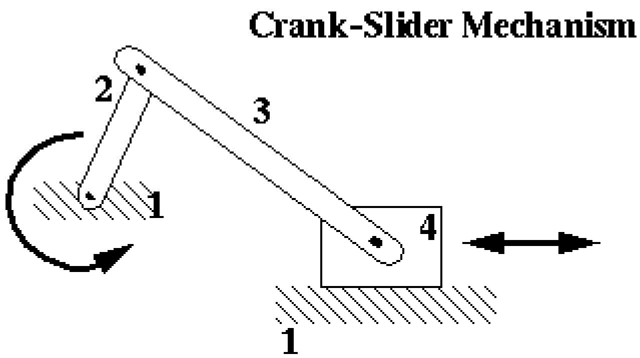
So this mechanism has 2 DOF.

The simplest, pin-jointed, single DOF mechanism is a four-bar mechanism. Depending on the lengths of the various links, you can have a double crank in which the two links pivoted to the ground make complete revolutions, you can have a crank-rocker mechanism in which one pivot to ground rotates and the other oscillates or you can have a double rocker where both links pivoted to ground oscillate. The coupler is the link connecting the two links which are pivoted to ground and this link both translates and oscillates or rotates.

4-bar Pin-Jointed Mechanism



Replacing the rocker of the four-bar mechanism with a full sliding joint results in a crank slider mechanism which should be a familiar mechanism.

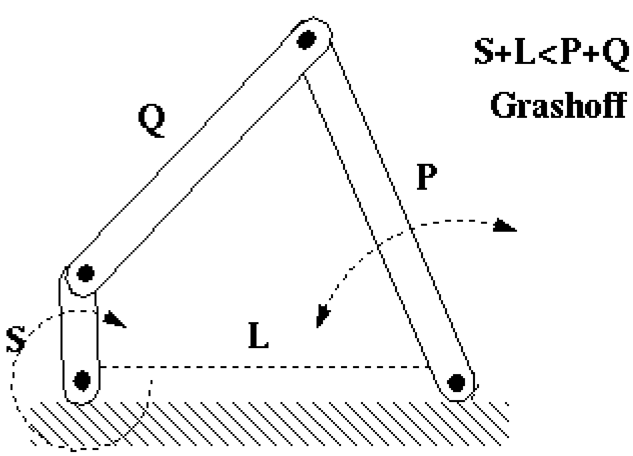


Grashof Condition

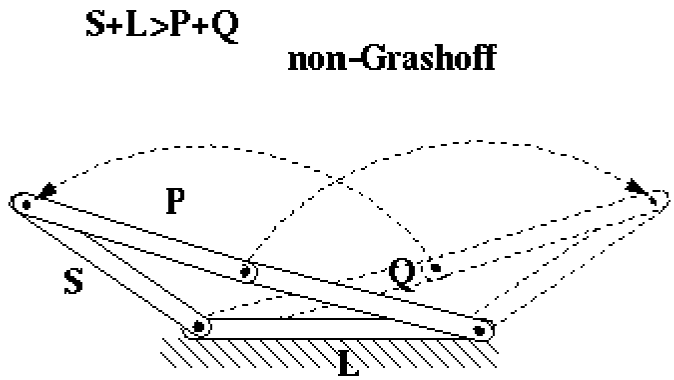
Without knowing how the links of a four-bar mechanism are connected, we can tell something about how it will behave just by knowing the lengths of the individual links and investigating the Grashof condition. Let the length of the shortest and longest links be denoted by **S** and **L**, respectively. The intermediate links will be labeled **P** and **Q**. If we compare the quantity **S**+**L** with **P**+**Q**, we can get a tell if any of the links will be able to rotate or not.

The Grashof condition states that if:

$S+L \leq P+Q$ then the mechanism is a Grashof mechanism and at least one link will be capable of a full revolution.



If $S+L>P+Q$ then the mechanism is non-Grashof and all combinations of the links will be double rockers and none of the links will be capable of a full rotation.



If S+L=P+Q then we have a special case Grashof and the mechanism will have changeovers where it can switch configurations and the output can be indeterminate.